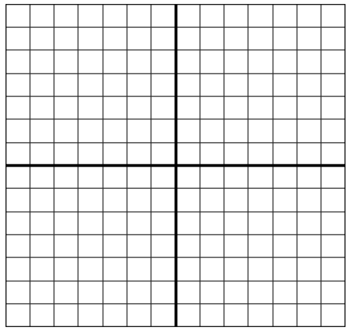
Unit 3: Determining (Increasing / Decreasing) Intervals
(Local Maximum / Minimum)

Name: ____________________

What is meant by increasing / decreasing intervals?

Increasing → positive slope (m = +) **Decreasing** → negative slope (m = –)

Ex: $f(x) = x^3 - 3x$



Increasing:

Decreasing:

Procedure: Identifying Increasing and Decreasing Intervals

1. Find the derivative of the given equation.
2. Find the *critical numbers.*
   - Any value of *x*, where $f'(x) = 0$
   - Any value of *x* that makes $f(x)$ or $f'(x)$ undefined.
3. Place those critical values on a number line to create test intervals.
4. Pick a test value from each interval and replace in $f'(x)$!
   - *f(x)* increases where $f'(x)$ is positive $\therefore$ (Slope is +)
   - *f(x)* decreases where $f'(x)$ is negative $\therefore$ (Slope is –)
5. Write your conclusion using ***interval notation***.

Examples:

1. Find the values of *x* where $f(x) = x^3 - 3x^2 - 9x + 5$ is increasing and decreasing.

Increasing: ____________________ Local Max: ____________________
Decreasing: ____________________ Local Min: ____________________

2. Find the intervals where $g(x) = \dfrac{x-1}{x^2}$ is increasing and decreasing.

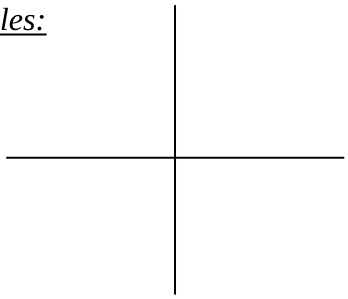
Increasing: ____________________ Local Max: ____________________
Decreasing: ____________________ Local Min: ____________________

Name: ___________________

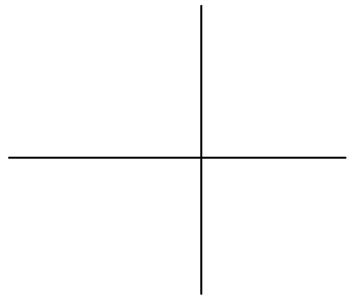
A graph has a **local maximum** at the point $(a, f(a))$ if the function is continuous at $x = a$ and the graph switches from increasing to decreasing at the critical value $x = a$.
These are also referred to as ______________________________.

A graph has a **local minimum** at the point $(a, f(a))$ if the function is continuous at $x = a$ and the graph switches from decreasing to increasing at the critical value $x = a$.
These are also referred to as ______________________________.

Examples:



Local Maximum at x = -2
Local Minimum at x = 1



Local Maximum at x = 0
Local Minimum at x = -2 and x = 2

Procedure: Finding Local Maximums and Minimums

First Derivative Test

1. Find the increasing/decreasing intervals.
2. Locate the critical numbers where the function is continuous and the graph changes direction.
   - -It is a local maximum if the intervals switch from increasing to decreasing.
   - -It is a local minimum if the intervals switch from decreasing to increasing.
3. To find the location for the (local max/min) as a point, place the *x*-value into the **original function** to find the y-value.

Second Derivative Test *NOTE: (Alternate option for determining Local Extrema!)*

1. Find the critical numbers for the first derivative.
2. Evaluate the critical numbers in the second derivative,
   - -If f'' is negative, the critical number is a local maximum.
   - -If f'' is positive, the critical number is a local minimum,
   - -If f'' is zero or undefined, the test is inconclusive.
3. To find the location for the (local max/min) as a point, place the *x*-value into the **original function** to find the y-value.

Fermat's Theorem:

If f has a local maximum or minimum at c & $f'(c) = exists$, then $f'(c) = 0$.

NOTE: Not all critical numbers have to be a local maximum or minimum. WHY?

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Dangerous Examples to consider!

$f(x) = x^3$

$g(x) = \frac{1}{x^2}$